CBSE 2025 EXAMINATIONS
XII MATHEMATICS (041)
Series Z6YWX ♦ Q.P. Code 65/6/1; 65/6/2; 65/6/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

☑ Q.P. Code - 65/6/1

SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is

(a) $n \times n$ (b) $n \times m$ (c) $m \times m$ (d) $m \times n$

Sol. (b) Let order of B be $p \times s$. So, the order of B' will be $s \times p$.

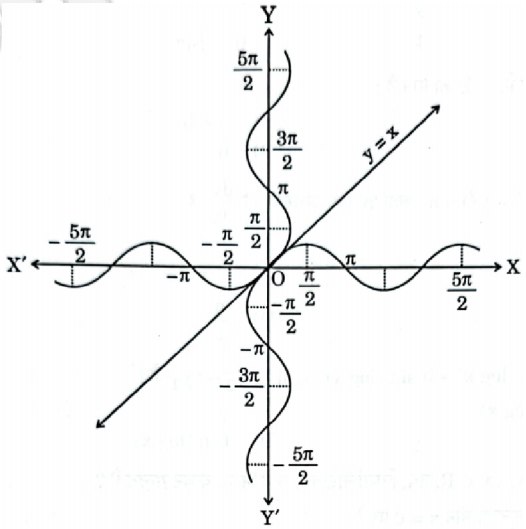
As AB' is defined so, $m = s$; also $B'A$ is defined so, $p = n$. Clearly, order of B will be $n \times m$.

02. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A is a/an

(a) scalar matrix (b) identity matrix (c) symmetric matrix (d) skew-symmetric matrix

Sol. (c) Note that, $A^T = A$. So, matrix A is a symmetric matrix.

03. The following graph is a combination of



(a) $y = \sin^{-1} x$ and $y = \cos^{-1} x$ (b) $y = \cos^{-1} x$ and $y = \cos x$

(c) $y = \sin^{-1} x$ and $y = \sin x$ (d) $y = \cos^{-1} x$ and $y = \sin x$

Sol. (c) $y = \sin^{-1} x$ and $y = \sin x$.

04. Sum of two skew-symmetric matrices of same order is always a/an

(a) skew-symmetric matrix (b) symmetric matrix

(c) null matrix (d) identity matrix

Sol. (a) Let A and B be two skew-symmetric matrices, so $A = -A^T$, $B = -B^T$.

Let $P = A + B$. Then $P^T = (A+B)^T = A^T + B^T = -A - B = -(A+B) = -P$.

Hence, $P = A + B$ is also a skew-symmetric matrix.

05. $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to

(a) $\frac{11\pi}{12}$ (b) $\frac{5\pi}{12}$ (c) $-\frac{5\pi}{12}$ (d) $\frac{7\pi}{12}$

Sol. (d) $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right] = \left[\frac{3\pi}{4} - \frac{\pi}{6}\right] = \frac{14\pi}{24} = \frac{7\pi}{12}$.

06. If $f(x) = \begin{cases} \frac{\log(1+ax) + \log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(a) a (b) $a + b$ (c) $a - b$ (d) b

Sol. (c) As f is continuous at $x = 0$, so $\lim\limits_{x\to 0} f(x) = f(0)$.

So, $\lim\limits_{x\to 0} \frac{\log(1+ax) + \log(1-bx)}{x} = k$

$\Rightarrow k = \lim\limits_{x\to 0}\left[\frac{\log(1+ax)}{ax} \times a + \frac{\log\{1+(-bx)\}}{(-bx)} \times (-b)\right]$ $[\because x \to 0 \Rightarrow ax \to 0, -bx \to 0$

$\Rightarrow k = [1 \times a + 1 \times (-b)] = a - b$.

07. If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is

(a) $\frac{x}{y}$ (b) $-\frac{x}{y}$ (c) $\frac{a}{x}$ (d) $\frac{a}{y}$

Sol. (a) Rewriting $\tan^{-1}(x^2 - y^2) = a$, we get $(x^2 - y^2) = \tan a$

On differentiating w.r.t. x, we get $2x - 2y \times \frac{dy}{dx} = 0 \quad \therefore \frac{dy}{dx} = \frac{x}{y}$.

08. If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2 y_2 + x y_1$ is

(a) $\cot(\log x)$ (b) y (c) $-y$ (d) $\tan(\log x)$

Sol. (c) $y = a\cos(\log x) + b\sin(\log x)$ ...(i)

On differentiating w.r.t. x, we get $\frac{dy}{dx} = -a\sin(\log x) \times \frac{1}{x} + b\cos(\log x) \times \frac{1}{x}$

$\Rightarrow x y_1 = -a\sin(\log x) + b\cos(\log x)$

Again differentiating w.r.t. x, we get $x y_2 + y_1 \times 1 = -a\cos(\log x) \times \frac{1}{x} - b\sin(\log x) \times \frac{1}{x}$

$\Rightarrow x^2 y_2 + y_1 \times x = -a\cos(\log x) - b\sin(\log x)$

By (i), we get $x^2 y_2 + x y_1 = -y$.

09. Let $f(x) = |x|$, $x \in \mathbb{R}$. Then, which of the following statement is **incorrect**?

(a) f has a minimum value at $x = 0$ (b) f has no maximum value in $\mathbb{R}$

(c) f is continuous at $x = 0$ (d) f is differentiable at $x = 0$

Sol. (d) "f is differentiable at $x = 0$" is **incorrect** statement.

You can check $Lf'(0) = -1 \neq Rf'(0) = 1$.

10. Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$. Then, $f(x)$ is

(a) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (b) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$

(c) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (d) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

Sol. (b) As $f'(x) = \frac{d}{dx}(f(x)) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$

$\Rightarrow d(f(x)) = \left(3(x^2 + 2x) - \frac{4}{x^3} + 5\right)dx$

On integrating, we get $\int d(f(x)) = \int \left(3(x^2 + 2x) - \frac{4}{x^3} + 5\right)dx$

$\Rightarrow f(x) = 3\left(\frac{x^3}{3} + x^2\right) + \frac{2}{x^2} + 5x + C = x^3 + 3x^2 + \frac{2}{x^2} + 5x + C$

Given that $f(1) = 0$, $0 = 1^3 + 3 \times 1^2 + \frac{2}{1^2} + 5 \times 1 + C \quad \Rightarrow C = -11$

Hence, $f(x) = x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$.

11. $\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to

(a) $\log(x+6) + C$ (b) $e^x + C$ (c) $\frac{e^x}{x+6} + C$ (d) $-\frac{1}{(x+6)^2} + C$

Sol. (c) $\int \frac{x+5}{(x+6)^2} e^x \, dx = \int \left[\frac{1}{(x+6)} + \frac{-1}{(x+6)^2}\right] e^x \, dx = \frac{e^x}{(x+6)} + C$.

Here $f(x) = \frac{1}{(x+6)}$, $f'(x) = -\frac{1}{(x+6)^2}$ and we have used $\int [f(x) + f'(x)] e^x \, dx = e^x f(x) + C$.

12. The order and degree of the differential equation $-\frac{d^4y}{dx^4} + 2e^{\frac{dy}{dx}} + y^2 = 0$ are, respectively

(a) –4, 1 (b) 4, not defined (c) 1, 1 (d) 4, 1

Sol. (b) Order : 4, degree : not defined.

13. The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is

(a) $3e^{4y} + 4e^{-3x} + C = 0$ (b) $e^{3x+4y} + C = 0$

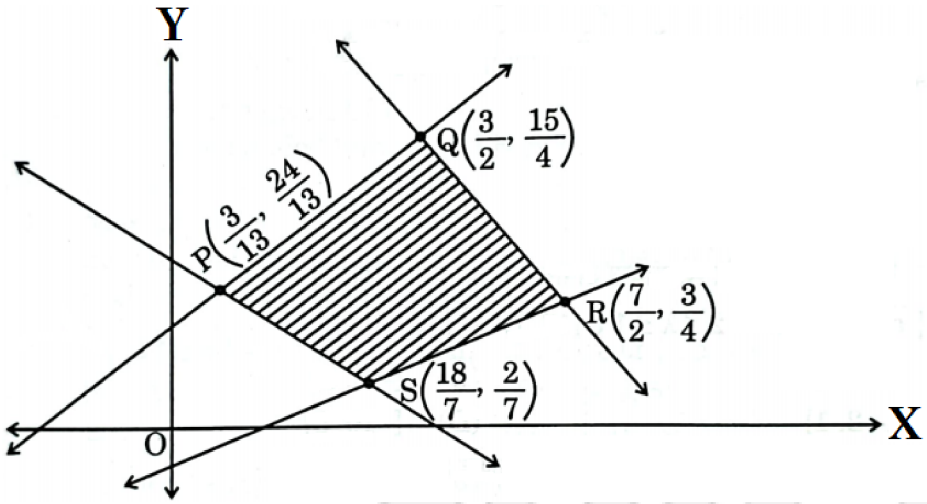
(c) $3e^{-3y} + 4e^{4x} + 12C = 0$ (d) $3e^{-4y} + 4e^{3x} + 12C = 0$

Sol. (d) $\log\left(\frac{dy}{dx}\right) = 3x + 4y \quad \Rightarrow \frac{dy}{dx} = e^{3x+4y} = e^{3x} \times e^{4y}$

$\Rightarrow \int e^{-4y}dy = \int e^{3x}dx$

$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$ i.e., $\frac{e^{-4y}}{4} + \frac{e^{3x}}{3} + C = 0$ i.e., $3e^{-4y} + 4e^{3x} + 12C = 0$.

14. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



(Note : The figure is not to scale.)

$P = \left(\frac{3}{13}, \frac{24}{13}\right)$, $Q = \left(\frac{3}{2}, \frac{15}{4}\right)$, $R = \left(\frac{7}{2}, \frac{3}{4}\right)$, $S = \left(\frac{18}{7}, \frac{2}{7}\right)$

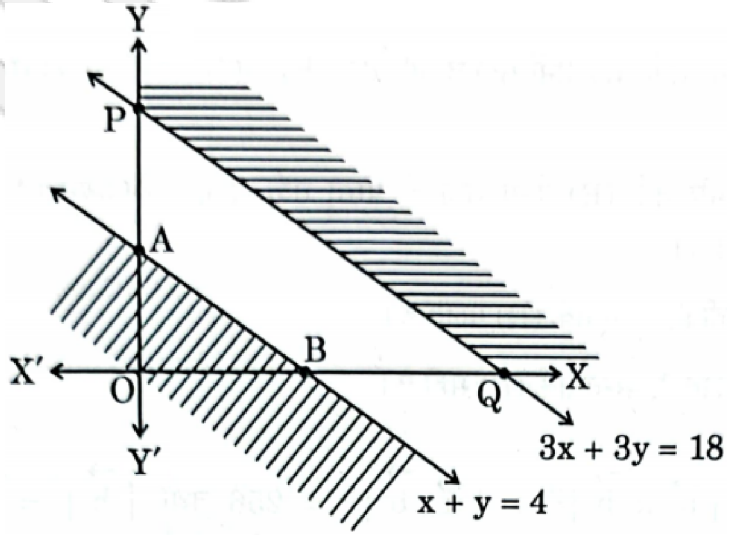
Which of the following statements is correct?

(a) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$ (b) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$

(c) (Value of Z at P) > (Value of Z at Q) (d) (Value of Z at Q) < (Value of Z at R)

Sol. (a) Note that, $Z_P = \frac{51}{13}$, $Z_Q = 9$, $Z_R = 5$, $Z_S = \frac{22}{7}$. Clearly, Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$.

15. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximized under the following constraints : $x + y \le 4$, $3x + 3y \ge 18$; $x, y \ge 0$.
Study the graph and select the correct option.



(Note : The figure is not to scale.)

The solution of the given LPP
(a) lies in the shaded unbounded region
(b) lies in ΔAOB
(c) does not exist
(d) lies in the combined region of ΔAOB and unbounded shaded region

Sol. (c) The solution of the given LPP 'does not exist'. Note that, there is no common shaded region for the given constraints. That is, there is no feasible region for LPP and hence no solution.

16. Let $|\vec{a}| = 5$ and $-2 \le \lambda \le 1$. Then, the range of $|\lambda \vec{a}|$ is

(a) $[5, 10]$ (b) $[-2, 5]$ (c) $[-2, 1]$ (d) $[-10, 5]$

Sol. As $-2 \le \lambda \le 1$ implies, $0 \le |\lambda| \le 2$. Then $0|\vec{a}| \le |\lambda||\vec{a}| \le 2|\vec{a}|$ i.e., $0 \le |\lambda\vec{a}| \le 2 \times 5$ i.e., $0 \le |\lambda\vec{a}| \le 10$.

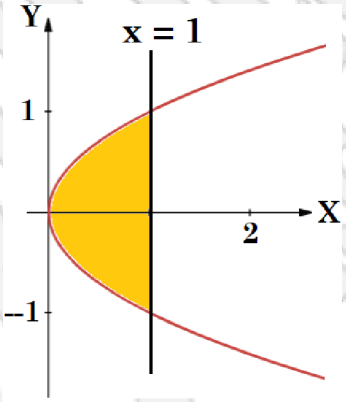
Hence, $0 \le |\lambda\vec{a}| \le 10$. Clearly, the range of $|\lambda \vec{a}|$ is $[0, 10]$.

Remark All the given options are incorrect.

17. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is

(a) $\frac{3}{2}$ Sq.units (b) $\frac{2}{3}$ Sq.units (c) 3 Sq.units (d) $\frac{4}{3}$ Sq.units

Sol. (d) Note that, $y^2 = x$ implies, $y = \pm\sqrt{x}$



$$\text{Required area} = 2\int_0^1 \sqrt{x}\, dx$$

$$= 2 \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1$$

$$= \frac{4}{3}\left[1^{\frac{3}{2}} - 0^{\frac{3}{2}}\right]$$

$$= \frac{4}{3} \text{ Sq.units}.$$

18. A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is

(a) $\frac{124}{125}$ (b) $\frac{1}{125}$ (c) $\frac{61}{125}$ (d) $\frac{64}{125}$

Sol. (c) Total number of pens : 15; out of which 3 are red pens and 12 are non-red pens.

$$\therefore \text{Required probability} = 1 - P(\text{no red pen was picked}) = 1 - \frac{12}{15} \times \frac{12}{15} \times \frac{12}{15} = 1 - \frac{64}{125} = \frac{61}{125}.$$

Direction : *Question numbers* ***19*** *and* ***20*** *are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** If $|\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R) : $\sin^2\theta + \cos^2\theta = 1$ and $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$ and $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

Sol. (a) Here both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Note that, $\left|\vec{a}\times\vec{b}\right|^2+\left|\vec{a}.\vec{b}\right|^2=256$ gives $\left[\left|\vec{a}\right|\left|\vec{b}\right|\sin\theta\right]^2+\left[\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right]^2=256$

$\Rightarrow\left[\left|\vec{a}\right|\times 8\sin\theta\right]^2+\left[\left|\vec{a}\right|\times 8\cos\theta\right]^2=256$

$\Rightarrow 64\left|\vec{a}\right|^2\left[\sin^2\theta+\cos^2\theta\right]=256$

$\Rightarrow\left|\vec{a}\right|^2\times 1=\dfrac{256}{64}=4 \quad \therefore\left|\vec{a}\right|=2.$

20. **Assertion (A) :** Let $f(x)=e^x$ and $g(x)=\log x$. Then $(f+g)x=e^x+\log x$, where domain of $(f+g)$ is $\mathbb{R}$.

Reason (R) : $\text{Dom.}(f+g)=\text{Dom.}(f)\cap\text{Dom.}(g)$.

Sol. (d) Here Assertion (A) is false and Reason (R) is true.

Note that, domain of $f(x)=e^x$ is $x\in\mathbb{R}$ and the domain of $g(x)=\log x$ is $x\in\mathbb{R}^+$.

Therefore, the $\text{Dom.}(f+g)=\text{Dom.}(f)\cap\text{Dom.}(g)=x\in\mathbb{R}^+$.

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. Find the domain of $f(x)=\sin^{-1}(-x^2)$.

Sol. For $\sin^{-1}(-x^2)$ to be defined, we must have $-x^2\in[-1,1]$

That is, $-1\le -x^2\le 1$ or, $-1\le x^2\le 1$

$\Rightarrow|x|\le 1 \qquad \therefore x\in[-1,1]$.

Therefore, domain of $f(x)=x\in[-1,1]$.

22. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x>0$.

OR

(b) If $(x)^y=(y)^x$, then find $\dfrac{dy}{dx}$.

Sol. (a) Let $y=\sqrt{e^{\sqrt{2x}}}$ and $z=e^{\sqrt{2x}}$.

Then $y=\sqrt{z}$

Differentiating w.r.t. z, we get $\dfrac{dy}{dz}=\dfrac{1}{2\sqrt{z}}$

Hence, $\dfrac{dy}{dz}=\dfrac{1}{2\sqrt{e^{\sqrt{2x}}}}$.

OR

(b) Given $(x)^y=(y)^x$

Taking log on both the sides, we get $\log(x)^y=\log(y)^x \quad \Rightarrow y\log(x)=x\log(y)$

On differentiating w.r.t. x, $y\times\dfrac{1}{x}+\log(x)\times\dfrac{dy}{dx}=x\times\dfrac{1}{y}\times\dfrac{dy}{dx}+\log(y)\times 1$

$\Rightarrow\left[\log(x)-\dfrac{x}{y}\right]\times\dfrac{dy}{dx}=\log(y)-\dfrac{y}{x}$

$\Rightarrow\left[\dfrac{y\log(x)-x}{y}\right]\times\dfrac{dy}{dx}=\dfrac{x\log(y)-y}{x}$

$\therefore\ \dfrac{dy}{dx}=\dfrac{y[x\log(y)-y]}{x[y\log(x)-x]}$.

23. Determine the values of x for which $f(x)=\dfrac{x-4}{x+1}$, $x\neq -1$ is an increasing or a decreasing function.

Sol. Given $f(x)=\dfrac{x-4}{x+1}=1-\dfrac{5}{x+1}$, $x\neq -1$,

We have $f'(x)=0-5\left(-\dfrac{1}{(x+1)^2}\right)=\dfrac{5}{(x+1)^2}>0\ \forall\ x\in\mathbb{R}-\{-1\}$

Hence, f is strictly increasing function for all $x\in\mathbb{R}-\{-1\}$ i.e., $x\in(-\infty,-1)\cup(-1,\infty)$.

24. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that $BC=3BA$.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

Sol. (a) Let $\overrightarrow{OC}=\vec{r}$.

Since $BC=3BA$ implies, $\overrightarrow{OC}-\overrightarrow{OB}=3\left[\overrightarrow{OA}-\overrightarrow{OB}\right]$



Then $\vec{r}-\vec{b}=3\left[\vec{a}-\vec{b}\right]=3\vec{a}-3\vec{b}$

$\Rightarrow \vec{r}=3\vec{a}-2\vec{b}$.

Hence, $\overrightarrow{OC}=3\vec{a}-2\vec{b}$.

OR

(b) Since $\vec{r}$ is inclined at equal angles to the three axes so, $\alpha=\beta=\gamma$.

Using $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$, we get $3\cos^2\alpha=1$

$\Rightarrow \cos\alpha=\dfrac{1}{\sqrt{3}}$

Therefore, the required vector $(\vec{r})=|\vec{r}|\left[\cos\alpha\,\hat{i}+\cos\beta\,\hat{j}+\cos\gamma\,\hat{k}\right]$

$$=5\sqrt{3}\left[\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right]=5(\hat{i}+\hat{j}+\hat{k}).$$

25. Determine if the lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ intersect with each other.

Sol. If the lines intersect each other, then $(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$

On comparing the coefficients of $\hat{i}$, $\hat{j}$, $\hat{k}$ both sides, we get

$1+3\lambda=4+2\mu$ ...(i), $1-\lambda=0$ ...(ii), $-1=-1+3\mu$ ...(iii)

From (ii) and (iii), we get $\lambda=1$, $\mu=0$.

Since $\lambda=1$, $\mu=0$ satisfies equation (i).

Hence, the given lines intersect each other and their point of intersection is at $4\hat{i}-\hat{k}$.

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. Let $A=\begin{bmatrix}1\\4\\-2\end{bmatrix}$ and $C=\begin{bmatrix}3&4&2\\12&16&8\\-6&-8&-4\end{bmatrix}$ be two matrices. Then, find the matrix B, if $AB=C$.

Sol. Let $B=\begin{bmatrix}x & y & z\end{bmatrix}$.

Now $AB = C$ gives, $\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}\begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x & y & z \\ 4x & 4y & 4z \\ -2x & -2y & -2z \end{bmatrix} = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$

By equality of matrices, we get $x = 3,\ y = 4,\ z = 2$.

Hence, $B = \begin{bmatrix} 3 & 4 & 2 \end{bmatrix}$.

27. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

OR

(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

Sol. (a) Put $x = \sin\theta \Rightarrow \theta = \sin^{-1} x$. As $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, then $-\frac{1}{2} \le \sin\theta \le \frac{1}{2}$ i.e., $-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$.

Now $y = \sin^{-1}(3x - 4x^3) = \sin^{-1}(3\sin\theta - 4\sin^3\theta) = \sin^{-1}(\sin 3\theta) = 3\theta$ $\left[\because -\frac{\pi}{2} \le 3\theta \le \frac{\pi}{2}\right.$

$\Rightarrow y = 3\sin^{-1} x$

$\therefore \frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$.

OR

(b) Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$. As $x \in (0, 1)$, then $0 < \tan\theta < 1$ i.e., $0 < \theta < \frac{\pi}{4}$.

Now $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\cos 2\theta) = 2\theta$ $\left[\because 0 < 2\theta < \frac{\pi}{2}\right.$

$\Rightarrow y = 2\tan^{-1} x$

$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$.

28. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41;\ x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, \text{ if } n \text{ is even} \\ n+1, \text{ if } n \text{ is odd} \end{cases}$ is a bijection.

Sol. (a) $2x + y = 41$ i.e., $x = \frac{41-y}{2}$, where $x, y \in N$.

The pair of values (x, y) in the relation R *(say)* will be given as :

$R = \{(20, 1), (19, 3), (18, 5), (17, 7), (16, 9), (15, 11), (14, 13), (13, 15), (12, 17), (11, 19),$
$(10, 21), (9, 23), (8, 25), (7, 27), (6, 29), (5, 31), (4, 33), (3, 35), (2, 37), (1, 39)\}$

Clearly, the domain $= \{1, 2, 3, 4, ..., 20\}$; range $= \{1, 3, 5, 7, ..., 39\}$.

As $(a, a) \notin R \ \forall \ a \in N$, so R is not reflexive.

Also $(20, 1) \in R$ but $(1, 20) \notin R$. So, R is not symmetric.

Further $(20, 1) \in R$ and $(1, 39) \in R$ but $(20, 39) \notin R$. So, R is not transitive.

As an equivalence relation must be reflexive, symmetric and transitive, hence the relation R in consideration here is not equivalence.

OR

(b) Given $f: N \to N$, $f(n) = \begin{cases} n-1, \text{ if } n \text{ is even} \\ n+1, \text{ if } n \text{ is odd} \end{cases}$.

Suppose $f(x_1) = f(x_2)$.

Note that, if x_1 is odd and x_2 is even, then we will have $x_1 + 1 = x_2 - 1$ i.e., $x_2 - x_1 = 2$ which is impossible.

Similarly, the possibility of x_1 being even and x_2 being odd can also be ruled out, using the similar argument.

Therefore, both x_1 and x_2 must be either odd or even.

Suppose **if both x_1 and x_2 are odd:**

Then $f(x_1) = f(x_2) \quad \Rightarrow x_1 + 1 = x_2 + 1 \quad \Rightarrow x_1 = x_2$.

Similarly, **if both x_1 and x_2 are even:**

Then $f(x_1) = f(x_2) \quad \Rightarrow x_1 - 1 = x_2 - 1 \quad \Rightarrow x_1 = x_2$.

Thus, f is one-one.

Also, any odd number $2r+1$ in the codomain (N) is the image of $2r+2$ in the domain (N) and any even number $2r$ in the codomain (N) is the image of $2r-1$ in the domain (N).

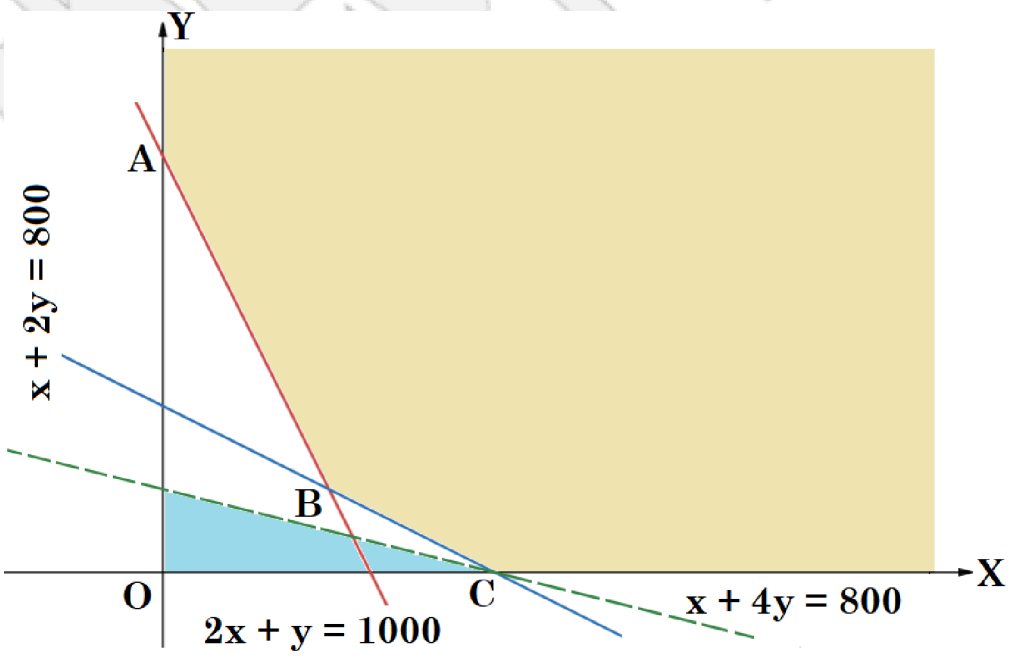
Thus, f is onto.

As f is both one-one and onto, hence it is a bijection.

29. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints $2x + y \geq 1000$, $x + 2y \geq 800$; $x, y \geq 0$.

Draw a neat graph of the feasible region and find the minimum value of Z.

Sol. Consider the graph shown below.



The corner points are $A(0, 1000)$, $B(400, 200)$ and $C(800, 0)$.

Corner Points	Value of Z
A(0, 1000)	$1\times0+4\times1000=4000$
B(400, 200)	$1\times400+4\times200=1200$
C(800, 0)	$1\times800+4\times0=800 \leftarrow$ Minimum

The minimum value of Z is 800. But since the feasible region is unbounded so, $Z=800$ may or may not be the minimum value. To check, let $x+4y<800$.

Since there is no common point between the feasible region of LPP and $x+4y<800$.

Hence, the minimum value of $Z=800$ and it is obtained at $(800, 0)$.

30. (a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A, B and C be $3\hat{i}-\hat{j}-2\hat{k}$, $\hat{i}+2\hat{j}-\hat{k}$ and $\hat{i}+5\hat{j}+3\hat{k}$ respectively. Find the vector and Cartesian equations of the line passing through A and parallel to line BC.

Sol. (a) Given P(2, 4, –1). Let M be foot of perpendicular drawn on the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$.

The direction ratios of the line are 1, 4, –9.

Any arbitrary point on the line is $M(\lambda-5, 4\lambda-3, -9\lambda+6)$.

Direction ratios of PM : $\lambda-7, 4\lambda-7, -9\lambda+7$.

Since PM is perpendicular to the given line.

So by using $a_1a_2+b_1b_2+c_1c_2=0$ for $\perp$ lines, we get

$1(\lambda-7)+4(4\lambda-7)+(-9)(-9\lambda+7)=0$

$\Rightarrow \lambda+16\lambda+81\lambda=7+28+63$

$\Rightarrow 98\lambda=98$

$\Rightarrow \lambda=1$

$\therefore M(-4, 1, -3)$ is the **foot of perpendicular** of the point P in line.

The required distance of point P from the line is, $PM=\sqrt{(-4-2)^2+(1-4)^2+(-3+1)^2}$

$\Rightarrow PM=\sqrt{36+9+4}=7$ units.

OR

(b) The direction ratios of line BC : $1-1, 5-2, 3-(-1)$ i.e., $0, 3, 4$.

Vector equation of line passing through A : $\vec{r}=3\hat{i}-\hat{j}-2\hat{k}+\lambda(3\hat{j}+4\hat{k})$.

Cartesian equation the line passing through A : $\frac{x-3}{0}=\frac{y+1}{3}=\frac{z+2}{4}$.

31. A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0.03 and that in committee II is 0.01, then find the probability that the person makes the correct decision of selection :

(i) in both committee.

(ii) in only one committee.

Sol. Let A and B denote the events of making a wrong selection in committee I and II respectively.

So, $P(A)=0.03 \quad \Rightarrow P(A')=1-0.03=0.97$

and $P(B)=0.01 \quad \Rightarrow P(B')=1-0.01=0.99$.

(i) Required probability $=P(A')\times P(B')=0.97\times0.99=0.9603$.

(ii) Required probability $=P(A')\times P(B)+P(A)\times P(B')=0.97\times0.01+0.03\times0.99=0.0394$.

SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. (a) Find $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$.

OR

(b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}dx$.

Sol. (a) Consider $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x+3}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$

$\Rightarrow x^2+1 = A(x-1)^2+B(x-1)(x+3)+C(x+3)$

On comparing the coefficients of like terms, we get $A=\frac{5}{8}$, $B=\frac{3}{8}$, $C=\frac{1}{2}$.

So, $\int \frac{x^2+1}{(x-1)^2(x+3)}dx = \frac{5}{8}\int \frac{1}{x+3}dx+\frac{3}{8}\int \frac{1}{x-1}dx+\frac{1}{2}\int \frac{1}{(x-1)^2}dx$

$= \frac{5}{8}\log|x+3|+\frac{3}{8}\log|x-1|+\frac{1}{2}\left[-\frac{1}{x-1}\right]+C$

$= \frac{5}{8}\log|x+3|+\frac{3}{8}\log|x-1|-\frac{1}{2(x-1)}+C.$

OR

(b) Let $I = \int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}dx$ …(i)

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{\cos x+\sin x}dx$ …(ii)

Adding (i) and (ii), we get $2I = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{1}{\cos x+\sin x}dx$

$\Rightarrow I = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)}dx = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)}dx$

$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\int_0^{\frac{\pi}{2}} \sec\left(x-\frac{\pi}{4}\right)dx = \frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right]_0^{\frac{\pi}{2}}$

$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right|-\log\left|\sec\left(-\frac{\pi}{4}\right)+\tan\left(-\frac{\pi}{4}\right)\right|\right]$

$$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\left[\log\left|\sqrt{2}+1\right|-\log\left|\sqrt{2}-1\right|\right]$$

$$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\left[\log\frac{\sqrt{2}+1}{\sqrt{2}-1}\right]=\frac{\pi}{4\sqrt{2}}\left[\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)\right]$$

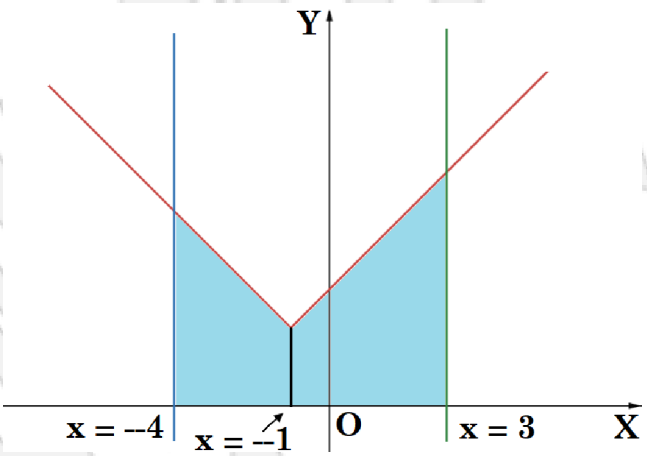
$$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\left[\log\left(\frac{(\sqrt{2}+1)^2}{2-1}\right)\right]=\frac{\pi}{4\sqrt{2}}\left[2\log(\sqrt{2}+1)\right]$$

Hence, $I=\frac{\pi}{2\sqrt{2}}\left[\log(\sqrt{2}+1)\right]$.

33. Draw a rough sketch for the curve $y=2+|x+1|$. Using integration, find the area of the region bounded by the curve $y=2+|x+1|$, $x=-4$, $x=3$ and $y=0$.

Sol. Here $y=2+|x+1|=\begin{cases}2+(x+1), \text{ if } x\geq -1\\ 2-(x+1), \text{ if } x<-1\end{cases}$

$$\Rightarrow y=\begin{cases}3+x, \text{ if } x\geq 1\\ 1-x, \text{ if } x<1\end{cases}.$$



$$\text{Required area}=\int_{-4}^{-1}(1-x)dx+\int_{-1}^{3}(3+x)dx$$

$$=-\frac{1}{2}\left[(1-x)^2\right]_{-4}^{-1}+\frac{1}{2}\left[(3+x)^2\right]_{-1}^{3}$$

$$=-\frac{1}{2}[4-25]+\frac{1}{2}[36-4]$$

$$=\frac{21}{2}+\frac{32}{2}$$

$$=\frac{53}{2}\text{ Sq. units}.$$

34. (a) Solve the differential equation $x^2y\,dx-(x^3+y^3)dy=0$.

OR

(b) Solve the differential equation $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ subject to the initial condition $y(0)=0$.

Sol. (a) Rewriting the D.E., we get $\frac{dx}{dy}=\frac{x^3+y^3}{x^2y}$

$$\Rightarrow \frac{dx}{dy}=\frac{x}{y}+\left(\frac{y}{x}\right)^2$$

Put $x=vy\Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$.

So, $v+y\frac{dv}{dy}=v+\frac{1}{v^2}\Rightarrow y\frac{dv}{dy}=\frac{1}{v^2}$

$$\Rightarrow \int v^2dv=\int\frac{dy}{y}$$

$\Rightarrow \frac{v^3}{3} = \log|y| + k$

$\Rightarrow \frac{x^3}{y^3} = 3\log|y| + C$, where $C = 3k$.

OR

(b) Rewriting the D.E., we get $\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$

On comparing with $\frac{dy}{dx} + P(x)\,y = Q(x)$, we get $P(x) = \frac{2x}{1+x^2}$, $Q(x) = \frac{4x^2}{1+x^2}$

Integration Factor $= e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1+x^2$.

Solution of D.E. is given by $(1+x^2)\times y = \int (1+x^2)\times \frac{4x^2}{1+x^2}dx + C$

$\Rightarrow (1+x^2)\times y = \int 4x^2\,dx + C$

$\Rightarrow (1+x^2)\times y = \frac{4}{3}x^3 + C$

It is given that $y(0) = 0$ i.e., $y = 0$, $x = 0$. So, $(1+0^2)\times 0 = \frac{4}{3}\times 0^3 + C \quad \Rightarrow C = 0$.

Hence, the required solution of D.E. is $(1+x^2)\times y = \frac{4}{3}x^3$ or $y = \frac{4x^3}{3(1+x^2)}$.

35. Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$. A point P(1, 6, 3), some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.

Sol. Rewriting the given line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$, we get $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ …(i)

Given P(1, 6, 3). Let M be foot of perpendicular drawn on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$.

The direction ratios of the line (i) are 1, 2, 3.

Any arbitrary point on the line (i) is $M(\lambda,\ 2\lambda+1,\ 3\lambda+2)$.

Direction ratios of PM : $\lambda-1, 2\lambda-5, 3\lambda-1$.

Since PM is perpendicular to the given line.

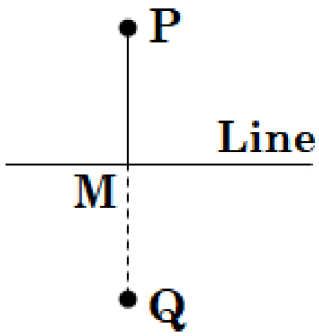
So by using $a_1a_2 + b_1b_2 + c_1c_2 = 0$ for $\perp$ lines, we get

$1(\lambda-1) + 2(2\lambda-5) + 3(3\lambda-1) = 0$

$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow \lambda = 1$

$\therefore$ M(1, 3, 5) is the **foot of perpendicular** of the point P in line.



Let image of point P in the line be $Q(h, p, s)$.

Since M is the mid-point of PQ, so $M\left(\frac{1+h}{2}, \frac{6+p}{2}, \frac{3+s}{2}\right) = M(1, 3, 5)$.

On comparing the respective coefficients, we obtain : Q(1, 0, 7) as the **image** of P in the line.

Also, the required distance $(PQ) = \sqrt{(1-1)^2 + (0-6)^2 + (7-3)^2} = \sqrt{0+36+16}$

$\Rightarrow PQ = \sqrt{52} = 2\sqrt{13}$ units.

Note that, we can also use $PQ = 2PM$ to find the required distance.

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. Three students, Neha, Rani and Sam go to a market to purchase stationary items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹90. Sam pays ₹70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions.

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix from $AX = B$.

(ii) Find $|A|$ and confirm if it is possible to find A^{-1}.

(iii) (a) Find A^{-1}, if possible, and write the formula to find X.

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix.

Sol. **(i)** Let the respective price of one pen, one notepad and one eraser be x, y and z (in ₹).

So, $4x + 3y + 2z = 60$, $2x + 4y + 6z = 90$, $6x + 2y + 3z = 70$.

Therefore, $\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$, where $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$.

(ii) $|A| = 4(12-12) - 3(6-36) + 2(4-24) = 0 + 90 - 40 = 50 \neq 0 \quad \therefore A^{-1}$ exists.

(iii) (a) Adjoint of matrix A, $\text{adj.}A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$.

So, inverse of matrix A is given as $A^{-1} = \dfrac{\text{adj.}A}{\det.A} = \dfrac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$.

Note that, $AX = B \quad \Rightarrow X = A^{-1}B$ [By pre-multiplication both sides with A^{-1}

That is, $X = \dfrac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$.

OR

(iii) (b) $A^2 - 8I = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow A^2 - 8I = \begin{bmatrix} 34 & 28 & 32 \\ 52 & 34 & 46 \\ 46 & 32 & 33 \end{bmatrix} - \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 26 & 28 & 32 \\ 52 & 26 & 46 \\ 46 & 32 & 25 \end{bmatrix}$.

Another possible solution can be obtained as below.

(i) Let the respective price of one pen, one notepad and one eraser be x, y and z (in ₹).

So, $4x + 3y + 2z = 60$, $2x + 4y + 6z = 90$, $6x + 2y + 3z = 70$.

Rewriting the system of equations : $4x + 3y + 2z = 60$, $x + 2y + 3z = 45$, $6x + 2y + 3z = 70$.

Therefore, $\begin{bmatrix}4&3&2\\1&2&3\\6&2&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}60\\45\\70\end{bmatrix}$, where $A=\begin{bmatrix}4&3&2\\1&2&3\\6&2&3\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $B=\begin{bmatrix}60\\45\\70\end{bmatrix}$.

(ii) $|A|=4(6-6)-3(3-18)+2(2-12)=0+45-20=25\neq 0$ $\therefore A^{-1}$ exists.

(iii) (a) Adjoint of matrix A, $\text{adj.}A=\begin{bmatrix}0&-5&5\\15&0&-10\\-10&10&5\end{bmatrix}$.

So, inverse of matrix A is given as $A^{-1}=\dfrac{\text{adj.}A}{\det.A}=\dfrac{1}{25}\begin{bmatrix}0&-5&5\\15&0&-10\\-10&10&5\end{bmatrix}$.

Note that, $AX=B \Rightarrow X=A^{-1}B$ [By pre-multiplication both sides with A^{-1}

That is, $X=\dfrac{1}{25}\begin{bmatrix}0&-5&5\\15&0&-10\\-10&10&5\end{bmatrix}\begin{bmatrix}60\\45\\70\end{bmatrix}$.

OR

(iii) (b) $A^2-8I=\begin{bmatrix}4&3&2\\1&2&3\\6&2&3\end{bmatrix}\begin{bmatrix}4&3&2\\1&2&3\\6&2&3\end{bmatrix}-8\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$\Rightarrow A^2-8I=\begin{bmatrix}31&22&23\\24&13&17\\44&28&27\end{bmatrix}-\begin{bmatrix}8&0&0\\0&8&0\\0&0&8\end{bmatrix}=\begin{bmatrix}23&22&23\\24&5&17\\44&28&19\end{bmatrix}$.

37. A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions.

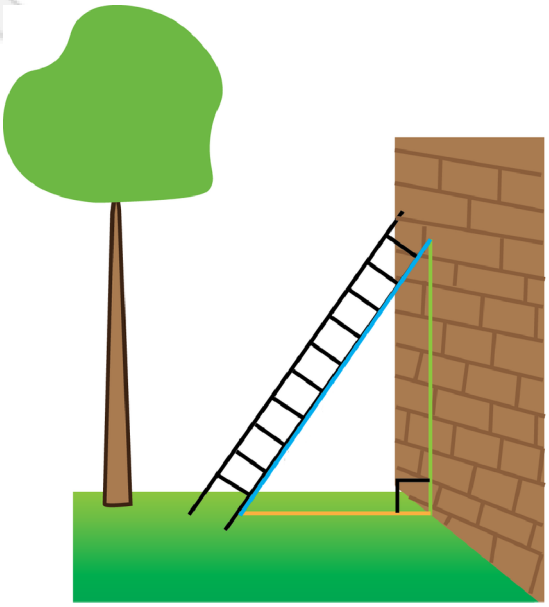
(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer.

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point.

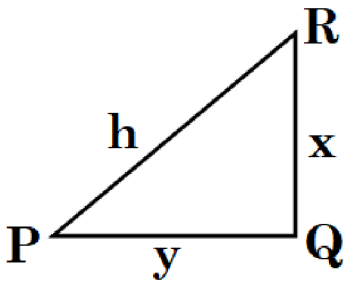
(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point.

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall?



Sol. **(i)** Consider the diagram shown at an instant.



Using Pythagoras theorem, we get

$PQ^2 + QR^2 = PR^2$

Therefore, $h^2 = x^2 + y^2$.

Area (A) of the right triangle PQR, $A = \frac{1}{2} \times PQ \times QR$

$\Rightarrow A = \frac{xy}{2} = \frac{x\sqrt{h^2 - x^2}}{2}$.

(ii) For $A = \frac{x\sqrt{h^2 - x^2}}{2}$, $\frac{dA}{dx} = \frac{1}{2}\left[x \times \frac{-2x}{2\sqrt{h^2 - x^2}} + \sqrt{h^2 - x^2} \times 1\right]$

$\Rightarrow \frac{dA}{dx} = \frac{1}{2}\left[\frac{-x^2 + h^2 - x^2}{\sqrt{h^2 - x^2}}\right] = \frac{1}{2}\left[\frac{h^2 - 2x^2}{\sqrt{h^2 - x^2}}\right]$

For critical point, $\frac{dA}{dx} = 0$, $\frac{1}{2}\left[\frac{h^2 - 2x^2}{\sqrt{h^2 - x^2}}\right] = 0$ $\Rightarrow h^2 - 2x^2 = 0$ $\Rightarrow x = \frac{h}{\sqrt{2}}$

$\therefore x = \frac{h}{\sqrt{2}}$ is the critical point.

(iii) (a) Using $\frac{dA}{dx} = \frac{1}{2}\left[\frac{h^2 - 2x^2}{\sqrt{h^2 - x^2}}\right]$, $\frac{d^2A}{dx^2} = \frac{1}{2}\left[\frac{\sqrt{h^2 - x^2}(-4x) - (h^2 - 2x^2) \times \frac{-x}{\sqrt{h^2 - x^2}}}{(\sqrt{h^2 - x^2})^2}\right]$

$\because \left(\frac{d^2A}{dx^2}\right)_{\text{at } x = \frac{h}{\sqrt{2}}} = \frac{1}{2}\left[\frac{\sqrt{h^2 - \frac{h^2}{2}}\left(-\frac{4h}{\sqrt{2}}\right) + (0)}{h^2 - \frac{h^2}{2}}\right] = \frac{1}{2}\left(-\frac{4}{\sqrt{2}}\right)\left[\frac{h \times h}{\sqrt{2}} \times \frac{2}{h^2}\right] = -2 < 0$

Hence, A is maximum at $x = \frac{h}{\sqrt{2}}$.

OR

(iii) (b) Using $h^2 = x^2 + y^2$, we get $5^2 = x^2 + y^2$

On differentiating w.r.t. t, we get $0 = 2x \times \frac{dx}{dt} + 2y \times \frac{dy}{dt}$

$\Rightarrow \frac{dx}{dt} = -\frac{y}{x} \times \frac{dy}{dt}$

When $y = 3$ m, then $5^2 = x^2 + 3^2$ $\Rightarrow x = 4$ m; also given that $\frac{dy}{dt} = -2$ m/s.

Hence, $\frac{dx}{dt} = -\frac{3}{4} \times (-2) = \frac{3}{2}$ m/s.

Hence, the height on the wall (x) is increasing at the rate of 1.5 m/s.

38. A shop selling electronic items sells smart phones of only three reputed companies A, B and C because chances of their manufacturing a defective smart phone are only 5%, 4% and 2% respectively. In his inventory he has 25% smart phones from company A, 35% smart phones from company B and 40% smart phones from company C.

A person buys a smart phone from this shop.

(i) Find the probability that it was defective.

(ii) What is the probability that this defective smart phone was manufactured by company B?

Sol. Let E_1, E_2 and E_3 be the events of smart phones from company A, B and C respectively.

Let E be the event that the smart phone is defective.

Now $P(E_1) = 25\%$, $P(E_2) = 35\%$, $P(E_3) = 40\%$;

Also $P(E \mid E_1) = 5\%$, $P(E \mid E_2) = 4\%$ and $P(E \mid E_3) = 2\%$.

(i) Using law of total probability, we get

$$P(E) = P(E_1) \times P(E \mid E_1) + P(E_2) \times P(E \mid E_2) + P(E_3) \times P(E \mid E_3)$$

$$\Rightarrow P(E) = \frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100} = \frac{125+140+80}{10000} = \frac{345}{10000} = \frac{69}{2000} = 0.0345$$

(ii) By using Bayes' theorem, we get $P(E_2 \mid E) = \dfrac{P(E_2) \times P(E \mid E_2)}{P(E)}$

$$\Rightarrow P(E_2 \mid E) = \frac{\frac{35}{100} \times \frac{4}{100}}{\frac{345}{10000}} = \frac{35 \times 4}{345} = \frac{7 \times 4}{69} = \frac{28}{69}.$$

☑ Q.P. Code - 65/6/2

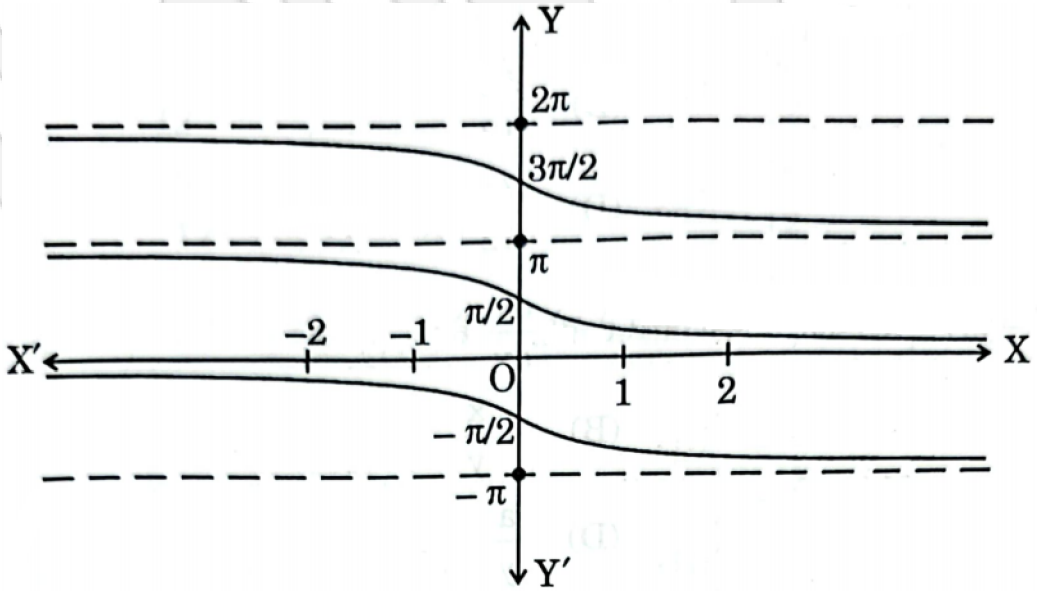
02. If $A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}$, then A is a

(a) null matrix (b) symmetric matrix

(c) skew-symmetric matrix (d) diagonal matrix

Sol. (c) A is a skew symmetric matrix. Since $A^T = -A$.

03. The graph shown below depicts



(a) $y = \cot x$ (b) $y = \cot^{-1} x$ (c) $y = \tan x$ (d) $y = \tan^{-1} x$

Sol. (b) The graph depicts $y = \cot^{-1} x$.

09. Let $f(x) = x^2$, $x \in \mathbb{R}$. Then, which of the following statements is **incorrect**?

(a) Minimum value of f does not exist (b) There is no point of maximum value of f in $\mathbb{R}$

(c) f is continuous at $x = 0$ (d) f is differentiable at $x = 0$

Sol. (a) Since $f(x)=x^2\geq 0\ \forall\ x\in\mathbb{R}$ so, the minimum value of f is 0. Therefore, the statement given in option (a) is **incorrect**.

12. The order and degree of the differential equation $\frac{d^2y}{dx^2}+4\left(\frac{dy}{dx}\right)=x\log\left(\frac{d^2y}{dx^2}\right)$ are respectively

(a) 0, 3 (b) 2, 1 (c) 2, not defined (d) 1, not defined

Sol. (c) 2, not defined.

18. Chances that three persons A, B and C go to the market are 30%, 60% and 50% respectively. The probability that at least one will go to the market is

(a) $\frac{14}{10}$ (b) $\frac{43}{50}$ (c) $\frac{9}{100}$ (d) $\frac{7}{50}$

Sol. (b) Required probability $=1-P(\text{none goes to the market})$

$$=1-P(\bar{A})\times P(\bar{B})\times P(\bar{C})$$

$$=1-\frac{70}{100}\times\frac{40}{100}\times\frac{50}{100}=1-\frac{14}{100}=\frac{86}{100}=\frac{43}{50}.$$

23. Determine those values of x for which $f(x)=\frac{2}{x}-5,\ x\neq 0$ is increasing or decreasing.

Sol. For $f(x)=\frac{2}{x}-5,\ f'(x)=-\frac{2}{x^2}$.

Note that, $\frac{2}{x^2}>0\ \forall\ x\in\mathbb{R}-\{0\}$. So, $f'(x)=-\frac{2}{x^2}<0\ \forall\ x\in\mathbb{R}-\{0\}$.

Hence f is strictly decreasing in $\mathbb{R}-\{0\}$.

25. Find the value of λ, if the following lines are perpendicular to each other :

$$l_1:\frac{1-x}{-3}=\frac{3y-2}{2\lambda}=\frac{z-3}{3};\ l_2:\frac{x-1}{3\lambda}=\frac{1-y}{1}=\frac{2z-5}{3}.$$

Sol. Rewriting the given lines, we get

$$l_1:\frac{x-1}{3}=\frac{y-\frac{2}{3}}{\frac{2\lambda}{3}}=\frac{z-3}{3};\ l_2:\frac{x-1}{3\lambda}=\frac{y-1}{-1}=\frac{z-\frac{5}{2}}{\frac{3}{2}}.$$

The d.r.'s of the respective lines are $3,\frac{2\lambda}{3},3;\ 3\lambda,-1,\frac{3}{2}$.

Using $a_1a_2+b_1b_2+c_1c_2=0$ for $\perp$ lines, we get $3\times 3\lambda+\frac{2\lambda}{3}(-1)+3\times\frac{3}{2}=0$

$$\Rightarrow\frac{27\lambda-2\lambda}{3}=-\frac{9}{2}$$

$$\Rightarrow 50\lambda=-27$$

$$\therefore\ \lambda=-\frac{27}{50}.$$

26. If $A=\begin{bmatrix}1 & -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}2 & 0 & 1\\ -1 & 3 & 4\\ 0 & 5 & 1\end{bmatrix}$ and $C=\begin{bmatrix}2\\3\\4\end{bmatrix}$, are three matrices, then find ABC.

Sol. $ABC=\begin{bmatrix}1 & -1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\ -1 & 3 & 4\\ 0 & 5 & 1\end{bmatrix}\begin{bmatrix}2\\3\\4\end{bmatrix}$

$$\Rightarrow ABC = \begin{bmatrix}3 & -3 & -3\end{bmatrix}\begin{bmatrix}2\\3\\4\end{bmatrix} = \begin{bmatrix}6-9-12\end{bmatrix}$$

$\therefore ABC = [-15]$.

31. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails. Hence, find the mean of the distribution.

Sol. Let p = probability of getting tail, q = probability of getting head on the coin.

Since $q = 3p$. So, using $p+q=1$ we have $p+3p=1$

$\Rightarrow p = \frac{1}{4}$, $q = 1-p = \frac{3}{4}$.

Let X : number of tails. So, X can take values 0, 1, 2 and 3.

Now $P(X=0) = \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$, $P(X=1) = 3\times\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$, $P(X=2) = 3\times\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4} = \frac{9}{64}$,

$P(X=3) = \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{64}$.

The table of probability distribution is given below.

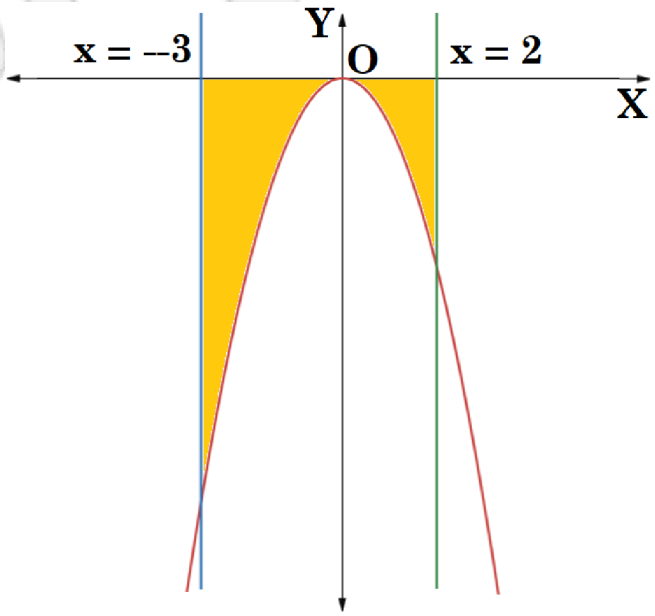
X	0	1	2	3
P(X)	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Now the mean $= \sum X P(X) = 0\times\frac{27}{64} + 1\times\frac{27}{64} + 2\times\frac{9}{64} + 3\times\frac{1}{64} = \frac{48}{64} = \frac{3}{4}$.

33. Use integration to find the area of the region enclosed by curve $y = -x^2$ and the straight lines $x = -3$, $x = 2$ and $y = 0$. Sketch a rough figure to illustrate the bounded region.

Sol. Given $y = -x^2$.

Refer the diagram shown.



$$\text{Required area} = \left|\int_{-3}^{2} -x^2\,dx\right|$$

$$= \left|-\frac{1}{3}\left[x^3\right]_{-3}^{2}\right|$$

$$= \left|-\frac{1}{3}\left[2^3-(-3)^3\right]\right|$$

$$= \left|-\frac{1}{3}\left[8-(-27)\right]\right|$$

$$= \left|-\frac{1}{3}\times 35\right|$$

$$= \frac{35}{3} \text{ Sq. units}.$$

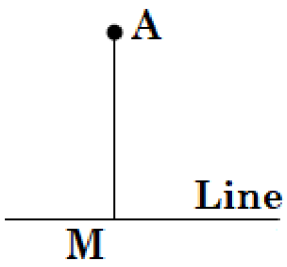
35. Find the foot of the perpendicular drawn from point (2, –1, 5) to the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$.

Also, find the length of the perpendicular.

Sol. Let A(2, –1, 5). For the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}=\lambda$, the direction ratios are 10, –4, –11.

Assume that M be the foot of perpendicular on the line drawn from A.

Any arbitrary point on the line is $M(10\lambda+11, -4\lambda-2, -11\lambda-8)$.



Direction ratios of AM : $10\lambda+9, -4\lambda-1, -11\lambda-13$.

Since AM is perpendicular to the given line.

So by using $a_1a_2+b_1b_2+c_1c_2=0$ for $\perp$ lines, we get

$10(10\lambda+9)+(-4)(-4\lambda-1)+(-11)(-11\lambda-13)=0$

$\Rightarrow 100\lambda+16\lambda+121\lambda+90+4+143=0$

$\Rightarrow \lambda=-\frac{237}{237}=-1$.

$\therefore$ M(1, 2, 3) is the **foot of perpendicular** of the point A in line.

Also the length of the perpendicular is, $AM=\sqrt{(1-2)^2+(2+1)^2+(3-5)^2}$

$\Rightarrow AM=\sqrt{1+9+4}=\sqrt{14}$ units.

☑ Q.P. Code - 65/6/3

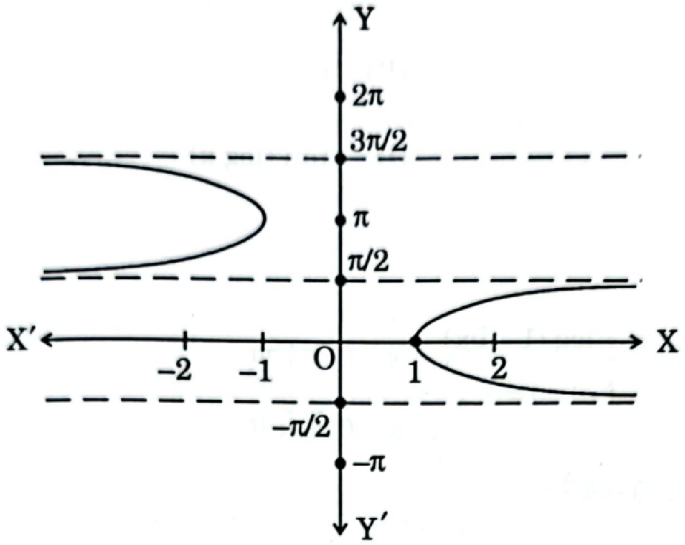
02. If $A=\begin{bmatrix}0 & 0 & -5\\ 0 & 3 & 0\\ 4.3 & 0 & 0\end{bmatrix}$, then A is a

(a) skew-symmetric matrix (b) scalar matrix

(c) diagonal matrix (d) square matrix

Sol. (d) A is a square matrix.

03. The graph shown below depicts



(a) $y=\sec^{-1}x$ (b) $y=\sec x$ (c) $y=\text{cosec}^{-1}x$ (d) $y=\text{cosec}\,x$

Sol. (a) The graph depicts $y=\sec^{-1}x$.

09. $f(x)=x^x$ has a critical point at

(a) $x=e$ (b) $x=e^{-1}$ (c) $x=0$ (d) $x=1$

Sol. (b) Note that, $f'(x)=x^x(1+\log_e x)$.

For the critical point, $f'(x)=0$, $x^x(1+\log_e x)=0$ i.e., $1+\log_e x=0$ [$\because x^x\neq 0$

So, $\log_e x = -1$ $\therefore x = e^{-1}$.

12. The order and degree of the differential equation $\left[\frac{d^2y}{dx^2} - 1\right]^2 = \frac{dy}{dx}$ are, respectively

(a) 2, 2 (b) 2, not defined (c) 1, 2 (d) 1, not defined

Sol. (a) Order : 2, degree : 2.

18. A meeting will be held only if all three members A, B and C are present. The probability that member A does not turn up is 0.10, member B does not turn up is 0.20 and member C does not turn up is 0.05. The probability of the meeting being cancelled is

(a) 0.35 (b) 0.316 (c) 0.01 (d) 0.65

Sol. (b) Probability that A does not turn up $= P(A') = 0.10$,

Probability that B does not turn up $= P(B') = 0.20$,

Probability that C does not turn up $= P(C') = 0.05$.

Since the meeting is cancelled if at least one of A, B, or C does not attend.

$\therefore$ Required probability of meeting being cancelled $= 1 - P(\text{meeting is held})$

$$= 1 - P(A \cap B \cap C)$$
$$= 1 - P(A).P(B).P(C)$$
$$= 1 - (0.90).(0.80).(0.95) = 1 - 0.684 = 0.316$$

23. Find the interval in which $f(x) = x + \frac{1}{x}$ is always increasing, $x \neq 0$.

Sol. For $f(x) = x + \frac{1}{x}$, $f'(x) = 1 - \frac{1}{x^2} = \left(1 - \frac{1}{x}\right)\left(1 + \frac{1}{x}\right)$

When $f'(x) = 0$, $\left(1 - \frac{1}{x}\right)\left(1 + \frac{1}{x}\right) = 0 \Rightarrow \left(1 - \frac{1}{x}\right) = 0, \left(1 + \frac{1}{x}\right) = 0$ $\therefore x = -1, 1$.

In $x \in (-\infty, -1] \cup [1, \infty)$, $f'(x) \geq 0$.

So, $f(x)$ is increasing in the interval $x \in (-\infty, -1] \cup [1, \infty)$.

25. Find the angle at which the given lines are inclined to each other :

$$l_1 : \frac{x-5}{2} = \frac{y+3}{1} = \frac{z-1}{-3}; \; l_2 : \frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-1}.$$

Sol. The direction ratios of lines l_1 and l_2 are 2, 1, –3 and 3, 2, –1 respectively.

Required angle between the lines, $\theta = \cos^{-1} \frac{2 \times 3 + 1 \times 2 + (-3) \times (-1)}{\sqrt{2^2 + 1^2 + (-3)^2}\sqrt{3^2 + 2^2 + (-1)^2}}$

$$\Rightarrow \theta = \cos^{-1} \frac{6 + 2 + 3}{\sqrt{4 + 1 + 9}\sqrt{9 + 4 + 1}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{11}{14}\right).$$

26. Find the value of x, if $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$.

Sol. $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$

$$\Rightarrow \begin{bmatrix} 1+2x+15 & 3+5x+3 & 2+x+2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 16+2x & 6+5x & 4+x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 16+2x+12+10x+4x+x^2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

By equality of matrices, we get $x^2+16x+28=0$

$\Rightarrow (x+14)(x+2)=0$

$\therefore x=-14,-2.$

31. Bag I contains 4 white and 5 black balls. Bag II contains 6 white and 7 black balls. A ball drawn randomly from bag I is transferred to bag II and then a ball is drawn randomly from bag II. Find the probability that the ball drawn is white.

Sol. Probability of transferring a white ball from bag I is $P(W_1)=\frac{4}{9}$;

Probability of transferring a black ball from bag I is $P(B_1)=\frac{5}{9}$.

Further the probability of drawing a white ball from bag II when a white ball was transferred from bag I is $P(W_2 \mid W_1)=\frac{7}{14}$;

Also the probability of drawing a white ball from bag II when a black ball was transferred from bag I is $P(W_2 \mid B_1)=\frac{6}{14}$.

Hence, by using law of total probability $P(W_2)=P(W_1)\times P(W_2 \mid W_1)+P(B_1)\times P(W_2 \mid B_1)$

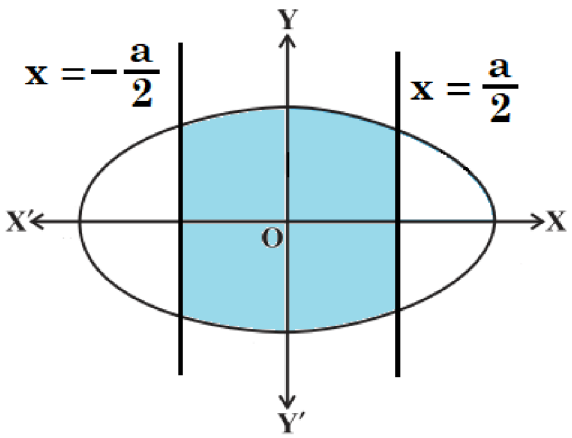
$$\Rightarrow P(W_2)=\frac{4}{9}\times\frac{7}{14}+\frac{5}{9}\times\frac{6}{14}=\frac{28+30}{126}=\frac{58}{126}=\frac{29}{63}.$$

33. Using integration, find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ bounded between the lines $x=-\frac{a}{2}$ to $x=\frac{a}{2}$.

Sol. We have $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \Rightarrow \frac{y^2}{b^2}=1-\frac{x^2}{a^2}$

$$\Rightarrow y^2=b^2\times\frac{a^2-x^2}{a^2}$$

$$\Rightarrow y=\pm\frac{b}{a}\sqrt{a^2-x^2}$$



$$\text{Required area}=2\times\frac{b}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}}\sqrt{a^2-x^2}\,dx$$

$$=4\times\frac{b}{a}\int_{0}^{\frac{a}{2}}\sqrt{a^2-x^2}\,dx$$

$$\Rightarrow \quad = \frac{4\times b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{\frac{a}{2}}$$

$$\Rightarrow \quad = \frac{4\times b}{a}\left[\left(\frac{a}{4}\sqrt{a^2-\frac{a^2}{4}}+\frac{a^2}{2}\sin^{-1}\frac{1}{2}\right)-0\right]$$

$$\Rightarrow \quad = \frac{4\times b}{a}\left[\left(\frac{a}{4}\times\frac{\sqrt{3}\,a}{2}+\frac{a^2}{2}\times\frac{\pi}{6}\right)\right] = \frac{4\times b}{a}\times\left(\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right)\times\frac{a^2}{4}$$

$$\Rightarrow \quad = a\,b\left(\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right)\text{ Sq.units}.$$

35. Show that the line passing through the points A(0, –1, –1) and B(4, 5, 1) intersects the line joining points C(3, 9, 4) and D(–4, 4, 4).

Sol. Equation of line AB : $\frac{x-0}{4-0}=\frac{y-(-1)}{5-(-1)}=\frac{z-(-1)}{1-(-1)}$ i.e., $\frac{x}{2}=\frac{y+1}{3}=\frac{z+1}{1}=\lambda$.

Equation of CD : $\frac{x-3}{-4-3}=\frac{y-9}{4-9}=\frac{z-4}{4-4}$ i.e., $\frac{x-3}{7}=\frac{y-9}{5}=\frac{z-4}{0}=\mu$.

The coordinates of random point on the lines AB and CD are respectively given as $P(2\lambda, 3\lambda-1, \lambda-1)$ and $Q(7\mu+3, 5\mu+9, 4)$.

If the lines AB and CD intersect, then points P and Q must coincide.

Therefore, on comparing the respective coordinates of points P and Q we get

$2\lambda-7\mu=3$ ...(i), $3\lambda-5\mu=10$ ...(ii), $\lambda=5$ ...(iii)

Solving (i) and (iii), we get $\lambda=5$, $\mu=1$.

Now LHS of (ii) : $3\lambda-5\mu=3\times5-5\times1=15-5=10=$ RHS of (ii).

Hence, the lines AB and CD intersect each other, point of intersection is (10, 14, 4).

With regards,
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The Series Z6YWX ♦ Q.P. Code 65/6/1; 65/6/2; 65/6/3 is of Panchkula, Chennai, Guwahati region.

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Kindly forward your concerns/feedbacks through **message** on **WhatsApp @ +919650350480** or mail at **iMathematicia@gmail.com**